Guideline for Design and Quality Control of Soil Improvement for Buildings
Deep and Shallow Cement Mixing Methods

National Institute for Land and Infrastructure Management
Architecture Research Institute

The Building Center of Japan
Chapter 5 Examination of Vertical Bearing Capacity of Improved Ground

5.1 Allowable Vertical Bearing Capacity of Improved Ground

(1) It shall be recognized that harmful deformation does not occur in structure by vertical load transmitted to the base of foundation slab.

(2) Vertical bearing capacity shall be decided by taking into account vertical bearing capacity of lower unimproved ground, peripheral friction force acting on improved ground, weight of improved ground and moderate safety factor.

[Explanation]
As for required vertical bearing capacity of improved ground, followings shall be examined.
1) Improved ground does not incur continuous ground settlement due to normal vertical load,
   2) Improved column does not incur harmful deformation due to normal vertical load,
   3) Improved ground does not incur harmful residual settlement due to vertical load during middle scale earthquake,
   4) Improved column does not incur harmful residual deformation due to vertical load during middle scale earthquake, and
   5) Improved ground does not incur failure during big scale earthquake.

Vertical load to the base of foundation slab shall be supported by resistance force of underground of improved ground and peripheral ground.(refer Fig.5.1.1) Also as shown in Fig.5.1.2, vertical load acting on the top of improved ground shall be supported by improved column and unimproved ground within the improved column.

The design method of vertical bearing capacity of improved ground shall be shown in Section 5.1 and vertical stress in improved column shall be shown in Section 5.2.
(1) **Design Contact Pressure at the Base of Foundation Slab**

It shall be recognized that allowable vertical bearing capacity of composit ground ($q_a$) can support safely design contact pressure ($\sigma_e$) at the base of foundation slab, i.e.,

$$\sigma_e = \frac{P}{A_f} \leq q_a \quad (5.1.1)$$

$\sigma_e$: design contact pressure (kN/m$^2$)
$P$: vertical load acting to the base of foundation slab (kN)
$A_f$: area of footing foundation or area of base of foundation slab (m$^2$)
$q_a$: allowable vertical bearing capacity of improved ground (kN/m$^2$)

When the load acting to the base of foundation slab is inclined and/or eccentric, design contact pressure shall be calculated by following equation.

$$\sigma_e = \frac{\alpha \cdot P}{A_f} \leq q_a \quad (5.1.2)$$

$\alpha$: contact pressure factor decided by the balance condition of force and moment due to load eccentricity and shape of base of foundation slab.

Magnitude of contact pressure factor differs from shape of base and functioning place of load to ground. Contact pressure derived from contact pressure factor is to be compared with allowable stress to vertical load neglecting effect of eccentric load.

(2) **Allowable Vertical Bearing Capacity ($q_a$) of Improved Ground**

Allowable bearing capacity of improved ground shall be to effective contact area of
the base of foundation. Long term allowable vertical bearing capacity \((q_{al})\) of composit ground, consisting of improved and unimproved ground, is calculated by Eq.(5.1.3), based on the ultimate vertical bearing capacity\((q_d)\) of lower ground and ultimate peripheral friction force \((\tau_d)\) of improved ground.\[Fig.5.1.3\]

Also vertical bearing capacity \((q_{a2})\) can be calculated by Eq.(5.1.4), or by summation of the value of point resistance of improved column and peripheral force of improved column. Long term allowable vertical bearing capacity of improved ground \(q_a\) shall be the smaller one of \(q_{a1}\) and \(q_{a2}\).\[Eq.(5.1.5)\]

\[
q_{a1} = \frac{1}{F_s} \left( q_d \cdot A_b + \sum (\tau_{di} \cdot h_i) \cdot L_s \right) / A_f \tag{5.1.3}
\]

\[
q_{a2} = \frac{1}{F_s} \left( n \cdot R_u \right) / A_f \tag{5.1.4}
\]

\[
q_a = \min (q_{a1}, q_{a2}) \tag{5.1.5}
\]

\(q_a\) : allowable vertical bearing capacity of improved ground (kN/m²)

\(q_{a1}\) : allowable vertical bearing capacity derived from mechanism of bearing capacity of composit ground (Fig.5.1.3) (kN/m²)

\(q_{a2}\) : allowable vertical bearing capacity derived from mechanism of vertical bearing capacity supposing that improved column supports independently (Fig.5.1.4) (kN/m²)

\(q_d\) : ultimate vertical bearing capacity of lower ground (kN/m²)

\(A_b\) : base area of improved ground (m²)

\(\tau_{di}\) : ultimate peripheral friction force functioning to periphery of improved ground(kN/m²)

\(h_i\) : thickness of layer (m)

\(L_s\) : peripheral length of improved ground(m)

An example of getting peripheral length shall be shown in Fig.5.1.5.
a) **Ultimate vertical bearing capacity of lower ground**

Ultimate vertical bearing capacity of lower ground shall be calculated by Eq. (5.1.6).

\[
q_d = i_c \cdot c N_c + i_q \cdot \gamma_1 B b N_\gamma + i_q \cdot \gamma_2 D_f^\gamma N_\gamma \\
i_c = i_q = \left[1 - \frac{\theta}{90^\circ}\right]^2 \quad i_\gamma = \left[1 - \frac{\theta}{\phi}\right]^2
\]  

(5.1.6)  

(5.1.7)

- \(q_d\): ultimate vertical bearing capacity of lower ground (kN/m\(^2\))
- \(\theta\): inclination angle of load (degree)
- \(\phi\): internal friction angle of lower ground (degree)
- \(\alpha, \beta\): coefficient depending on the figure of improved ground shown in Table 5.1.1
- \(B_b\): short side length of improved ground (m)
- \(L_b\): long side length of improved ground (m)
- \(c\): cohesion of lower ground (kN/m\(^2\))
- \(N_c, N_\gamma, N_\gamma\): bearing capacity factor (Table 5.1.2)
- \(\gamma_1\): density of lower ground (k N/m\(^3\))
  - submerged density for part under ground water level
- \(\gamma_2\): average density of ground above lower ground
  - submerged density for part under ground water level
- \(D_f^\gamma\): depth from ground surface to improved point

<table>
<thead>
<tr>
<th>Factor</th>
<th>Circle</th>
<th>Shape except circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>1.2</td>
<td>1.0+0.2 (\frac{B_b}{L_b})</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.3</td>
<td>0.5–0.2 (\frac{B_b}{L_b})</td>
</tr>
</tbody>
</table>
Table 5.1.2 Bearing capacity factor

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$N_c$</th>
<th>$N_r$</th>
<th>$N_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>5.1</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>5°</td>
<td>6.5</td>
<td>0.1</td>
<td>1.6</td>
</tr>
<tr>
<td>10°</td>
<td>8.3</td>
<td>0.4</td>
<td>2.5</td>
</tr>
<tr>
<td>15°</td>
<td>11.0</td>
<td>1.1</td>
<td>3.9</td>
</tr>
<tr>
<td>20°</td>
<td>14.8</td>
<td>2.9</td>
<td>6.4</td>
</tr>
<tr>
<td>25°</td>
<td>20.7</td>
<td>6.8</td>
<td>10.7</td>
</tr>
<tr>
<td>28°</td>
<td>25.8</td>
<td>11.2</td>
<td>14.7</td>
</tr>
<tr>
<td>32°</td>
<td>35.5</td>
<td>22.0</td>
<td>23.2</td>
</tr>
<tr>
<td>36°</td>
<td>50.6</td>
<td>44.4</td>
<td>37.8</td>
</tr>
<tr>
<td>40° 以上</td>
<td>75.3</td>
<td>93.7</td>
<td>64.2</td>
</tr>
</tbody>
</table>

The value of cohesion $c$ and internal friction angle $\phi$ shall be decided by taking into account the range of stress increase in underground induced by load. Inclined angle of load in Eq. (5.1.6) shall be deemed zero if thickness of improved soil is thin and improved soil does not move together with foundation.

Except above mentioned method, ultimate vertical bearing capacity of lower ground shall be estimated by plate loading test or Swedish sounding test. Equation (5.1.8) shows allowable vertical bearing capacity of lower ground $q_{e1}$ by plate loading test. Eq. (5.1.9) shows the same one $q_{e2}$ by Swedish sounding test. To use the equation of bearing capacity, type of building foundation, ground condition and range of stress increase in under ground shall be considered. In case of plate loading test, consideration shall be necessary for difference between foundation size and loading plate size. In case of Swedish sounding test, since $N_{sw}$ is average from improved bottom to 2 meters deep, other method shall be used if the range of stress increase in underground is different from the range.
\[ q_{el} = 3q_t/F_s + (1/3)N'\gamma_2 D_f \]  \hspace{1cm} (5.1.8)

\( q_{el} \): allowable vertical bearing capacity of lower ground by plate loading test  
\( F_s \): safety factor, 3 for general load, 1.5 for middle scale earthquake  
\( q_t \): smaller value of the two: 1/2 of yielding load by plate loading test, or 1/3 of ultimate bearing capacity  
\( N' \): factor decided by ground condition under the foundation (Table (5.1.3))  
\( D_f \): depth from ground surface to improved point  
\( \gamma_2 \): average density above lower ground, submerged density in water for the part under groundwater level

<table>
<thead>
<tr>
<th>Factor</th>
<th>Dense sandy ground</th>
<th>Sandy ground (except dense sand)</th>
<th>Clayey ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N' )</td>
<td>12</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ q_{e2} = (90 + 1.8N_{sw})/F_s \]  \hspace{1cm} (5.1.9)

\( q_{e2} \): allowable vertical bearing capacity of lower ground by Swedish sounding test  
\( F_s \): safety factor, 3 for general load, 1.5 for middle scale earthquake  
\( N_{sw} \): average half rotating number each meter by Swedish sounding of the depth from base of foundation to 2 meters deep (150 if exceeds 150)

**b) In case of 2 layers ground**

When soil layer of lower ground is not homogeneous, allowable vertical bearing capacity of lower ground shall be calculated considering the effect. An example of calculation is shown in Fig.5.1.8.
(3) Ultimate Peripheral Friction Force

Ultimate peripheral friction force of improved ground shall be calculated by Eq.(5.1.10) and Eq.(5.1.11) by each layer.

\[
\text{Clayey soil} \quad \tau_d = c \quad \text{or} \quad q_u/2 \\
\text{Sandy soil} \quad \tau_d = \frac{10N}{3} \quad (\text{kN/m}^2)
\]

\(c\): cohesion of clayey soil (kN/m\(^2\))
\(q_u\): unconfined compressive strength of clayey soil (kN/m\(^2\))
\(N\): \(N\) value of sandy soil

(4) Ultimate Vertical Bearing Capacity of Improved Column

Ultimate vertical bearing capacity of improved column can be obtained by loading test or by Eq.(5.1.11). [refer Fig.(5.1.4)]

Method of loading test applies vertical loading test of pile. Ultimate bearing capacity in loading test is load which amount of settlement at the top of column reaches to 10% of column diameter.

\[
R_a = R_{pu} + \phi \cdot \sum \tau_d \cdot h_i
\]

\(R_a\): ultimate vertical bearing capacity of improved column (kN)
\( R_{pu} \): ultimate vertical bearing capacity at the point of improved column (kN)

\( \phi \): peripheral length of improved column (degree)

\( \tau_{di} \): ultimate peripheral friction force (kN/m²)

\( h_i \): thickness of layer (m)

Ultimate vertical bearing capacity at the point of improved column shall be calculated by Eq.(5.1.13) and Eq.(5.1.14).

- Sandy soil: \( R_{pu} = 75 \cdot \overline{N} \cdot A_p \) (kN)  
  \( \text{(5.1.13)} \)
- Clayey soil: \( R_{pu} = 6c \cdot A_p \) (kN)  
  \( \text{(5.1.14)} \)

\( R_{pu} \): ultimate vertical bearing capacity at the point of improved column (kN)

\( \overline{N} \): average N value in the range of plus minus 1d from the point of improved column (d is minimum width of improved column)

\( c \): cohesion of clayey soil layer (kN/m²)

\( A_p \): effective area at the point of improved column (m²)

### 5.2 Vertical Stress in Improved Column

| (1) Load to the base of slab shall be supported by improved column and unimproved ground among improved column, and their share to support shall be decided by taking into account ground condition and improvement specification. |
| (2) Compressive stress of improved column shall be recognized to be less than allowable compressive stress of improved column. |

#### [Explanation]

**1) Calculation of Vertical Stress in Improved Column by the Load to the Base of Foundation Slab**

As shown in Fig.5.2.1, transmitted load from the base of foundation slab to improved column shall be shared by improved column and unimproved ground among improved columns. Vertical stress in the improved column shall be calculated considering stress concentration factor \( \mu_p \) which is ratio of stress in improved column and design contact pressure (Eq.5.2.1).

\[ q_p = \mu_p \cdot \sigma_e \]  
\( \text{(5.2.1)} \)

\( q_p \): vertical stress at the top of improved column (kN/m²)

\( \mu_p \): stress concentration factor

\( \sigma_e \): design contact pressure acting on the base of foundation slab (kN/m²)
(2) Stress Concentration Factor

As shown in Fig.5.2.1, load acting on the base of foundation slab shall be supported by reaction force of improved column and unimproved ground among improved columns. The stress concentration factor $\mu_p$ is calculated by Eq. (5.2.2).

$$\mu_p = \frac{n}{1 + (n - 1)a_p}$$  \hspace{1cm} (5.2.2)

$\mu_p$: stress concentration factor

$a_p$: improved ratio of the base of foundation slab

$n$: stress assignment ratio \hspace{1cm} n = q_p/q_s$

$q_p$: vertical stress at the top of improved column

$q_s$: ground reaction at unimproved ground among improved columns

$$a_p = \frac{\Sigma A_p}{A_f}$$  \hspace{1cm} (5.2.3)

$\Sigma A_p$: area of improved column at the base of foundation slab

$A_f$: area of foundation slab

Sharing ratio of stress shall be calculated by Eq. (5.2.4).

$$n = \frac{E_p(\lambda z + n z)}{E_s(\alpha \nu z \cdot \lambda z + n z \nu z)}$$  \hspace{1cm} (5.2.4)

$$\alpha \nu = \frac{(1 - \nu z)}{1 + \nu z \cdot (1 - 2\nu z)}$$  \hspace{1cm} (5.2.5)
\[ \lambda_i = \frac{H_i}{B_p}, \quad n_{12} = \frac{E_1}{E_z}, \quad n_{22} = \frac{E_2}{E_z} \]  

(5.2.6)

\( H_1 \): layer thickness of first layer and is equivalent to improved length (m)
\( H_2 \): layer thickness of second layer and is around equivalent to width of short side of improved column (m)
\( B_p \): width of short side of improved column (m)
\( E_p \): Young’s modulus of improved column (k N/m²)
\( E_1, E_2 \): Young’s modulus of first and second layer respectively
\( \nu_1 \): Poisson’s ratio of first layer and second layer
\( \nu_2 \): Respectively
\( \alpha \nu_1 \): increase ratio of Young’s modulus of vertical direction arisen from side confinement

Fig.5.2.2 Relation between improved column and unimproved ground among improved columns

When the improved column is supported by hard ground, sharing ratio of stress shall be given by Eq.(5.2.7).

\[ n = \frac{E_p}{E_p \cdot \alpha \nu_1} = \frac{n_{12}}{n_{22} \cdot \alpha \nu_1} \]  

(5.2.7)

When lower ground is hard, ratio of Young’s modulus with peripheral ground is less than 0.1, and ratio of Young’s modulus of lower ground with improved column is
more than 10, \( n \) in Eq.(5.2.7) will be more than 100. Therefore, Eq.(5.2.2) is approximated to Eq.(5.2.8).

\[
\mu_p = \frac{1}{n
\text{v}} \tag{5.2.8}
\]

(3) **Stress Check of Improved Column**

It shall be recognized that the vertical stress at the top of improved column (Eq.(5.2.1)) is less than allowable compressive stress of improved column. (Eq.(5.2.9)) and Eq.(5.2.10)

\[
q_p \leq f_c \tag{5.2.9}
\]

\( q_p \): vertical stress at the top of improved column \( (\text{kN/㎡}) \)

\( f_c \): allowable compressive stress of improved body \( (\text{kN/㎡}) \)

\[
f_c = \frac{1}{F_{sp}} F_c \tag{5.2.10}
\]

\( F_{sp} \): safety factor, equivalent to \( F_s \)

Vertical stress arisen in inside of improved column has a tendency to become smaller due to the peripheral friction force. Therefore, it is enough to examine the vertical stress at the top of column.

If there is possibility that the negative friction occurs at side of improved column, vertical stress lower portion of improved column shall be examined.

In case of getting stress in the improved column, it is necessary to check that stress at every portion is less than allowable stress at the compressive side of improved column, in static state and also in middle scale earthquake.
Chapter 6 Examination of Horizontal Bearing Capacity of Improved Ground

6.1 Horizontal Bearing Capacity in Static State and in Middle Scale Earthquake

It shall be recognized that improved ground does not arise harmful deformation to structure by lateral earth pressure and inertial force of structure during middle scale earthquake.

[Explanation]

It shall be recognized by following (1) and (2) that stress in improved column is less than allowable compressive stress, allowable tensile stress and allowable shearing stress by horizontal load which is arisen by lateral earth pressure and inertial force of structure during middle scale earthquake. Required capacity of improved ground to the horizontal load in static state and during middle scale earthquake is a stipulation concerning deformation. According to the experience until now, since harmful deformation to structure is hardly seen if stress in improved ground is less than present allowable stress, examination of allowable stress shall be alternative of examination of deformation.

As for examination method of horizontal bearing capacity of improved ground, there are other alternatives such as finite element method and calculation deeming caisson or elastic beam instead of improved column. These methods shall be allowed if these phenomena are well explained.

(1) Examination of Bending Stress by Reaction Force Method of Linear Elastic Ground

When cement deep mixing method is used for structure foundation, since it is classified into short pile, examination of bending stress shall be taken into account improved length. Calculation step is as follows.

(a) Calculation of load share to improved column
(b) Calculation of horizontal ground force factor of original ground
(c) Calculation of transformation factor of Improved column
(d) Calculation of bending stress and comparison of allowable stress

Fig.6.1.1 Examination steps of bending stress by linear elastic ground reaction force force
Examination methods and noticed points are as follows by step. In case of ratio of improved length by improved width is less than 1, examination of bending stress can be omitted. In observation of horizontal loading test, crack was not observed in improved ground with slenderness ratio of 1.75, and in improved wall type ground with slenderness ratio of 2.00 and 2.25.

Table 6.1.1 Load to the improved column

<table>
<thead>
<tr>
<th>Column style</th>
<th>Without lapping</th>
<th>Wall style</th>
<th>Lapping style</th>
<th>Block style</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uniform load to the foundation base</td>
<td>Calculation formula</td>
<td>Nonuniform load to the foundation base</td>
<td>Calculation formula</td>
</tr>
<tr>
<td></td>
<td>Conceptual figure.</td>
<td></td>
<td>Conceptual figure.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W_p = \frac{\pi \cdot d \cdot d_2}{W/n}$</td>
<td>$Q_p = k_a \cdot \frac{Q \cdot d_1 \cdot d_2}{Q/n}$</td>
<td>$W_p = \frac{\pi \cdot d \cdot d_2}{W/n}$</td>
<td>$Q_p = \frac{Q \cdot W}{W_0}$</td>
</tr>
<tr>
<td></td>
<td>$Q_a = k_a \cdot W_p$</td>
<td>$Q_a = k_a \cdot W_p$</td>
<td>$Q_a = k_a \cdot W_p$</td>
<td>$Q_a = k_a \cdot W_p$</td>
</tr>
<tr>
<td></td>
<td>$W_s = W$</td>
<td>$W_s = W$</td>
<td>$W_s = W \cdot d_1$</td>
<td>$W_s = W \cdot d_1$</td>
</tr>
<tr>
<td></td>
<td>$Q_s = Q$</td>
<td>$Q_s = Q$</td>
<td>$Q_s = Q \cdot d_1$</td>
<td>$Q_s = Q \cdot d_1$</td>
</tr>
</tbody>
</table>

$: Area which an improved column pays
a) Calculation of load share to improved column

Load to improved column is classified by combining of load distribution at foundation base and improved type (Table 6.1.1). Load to foundation base shall be shared by improved column and unimproved ground among improved columns, but improved column shall bear whole load in case of horizontal bearing capacity. Since load share of horizontal load is not well known, rule was prepared in safety side.

Calculation of load share to the improved column is as follows.

① Vertical load share to improved one column shall be calculated according to the load distribution to foundation base, and horizontal load shall be proportion to vertical load in case of column type. Examination shall be done to improved column which bears the maximum vertical load among columns.

② In case of wall type or block type, whole load shall be function to improved column as a group.

Arrangements of improved column have many types according to foundation type. In case of other cases except above table, way of thinking is the same.

b) Calculation of horizontal ground reaction factor of ground

Horizontal ground reaction factor shall be calculated from deformation factor as follows.

① When pitch of improved column is wide.

When pitch of improved column is bigger than 3 to width of improved column (d ≥ 3 \(b\)), horizontal ground reaction factor shall be estimated by Eq (6.1.1).

\[
k_h = \frac{1}{30} \cdot \alpha \cdot E_0 \cdot \left(\frac{b}{30}\right)^{3/4} \times 10^2 \tag{6.1.1}\]

\(k_h\): horizontal ground reaction factor (kN/m³)
\(\alpha\): factor (4)
\(E_0\): Young’s modulus of ground (kN/m²)
\(b_1\): width of column in direction to force acting (cm)

Young’s modulus of ground is as follows.

- from unconfined compressive strength \(E_s = 56 q_u\) (kN/m²)
- from standard penetration test \(E_s = 7 N \times 10^2\) (kN/m²)

Above Young’s modulus and unconfined compressive strength shall be equal under premise of \(q_u = N/8 \times 10^2\) (k N/m²). Moreover, relation between \(q_u\) and \(E_s\) is set to be in domain of experimental equation. Young’s modulus is used if it is derived from unconfined or triaxial compressive test.

\(k_h\) of Eq.(6.1.1) is to value of 1 cm deformation at the top of column, but \(k_{hy}\) to
arbitrary deformation (cm) shall be calculated by following equation. In this occasion repeating calculation is needed but the method is described in (3) in Section 6.1.

\[ k_{hp} = k_h \cdot y^{-1/2} \]

(2) When pitch of improved column is narrow.

When cement deep mixing method is used, since pitch of improved column \( d \) by improved column width \( b \) is small in general, column must be considered as well as pile. If \( d/b \) is smaller than 3, horizontal ground reaction force factor \( k_h' \) shall be calculated by next equation by taking into account factor on column group effect.

\[ k_h' = \mu \cdot k_h \]  \( (6.1.2) \)

\( k_h' \): horizontal ground reaction force factor taking into account column group effect (kN/m³)
\( k_h \): horizontal ground reaction force factor as single column by Eq.(6.1.1) (kN/m³)

\( \mu \) is factor on column group effect and next \( \mu_1, \mu_2 \) or \( \mu_{12} \) is used.

When \( d < 3b \), and \( d_2 \geq 3b_2 \), Fig.6.1.2(a)

\[ \mu_1 = 1 - 0.2(3 - R_1) \]

\( R_1 \): pitch of improved column \( d_1 \)/width of improved column \( b_1 \) \((<3)\)

\( \mu_1^* = (k_h \text{ derived from Eq.}(6.1.1) \text{ by using } b_1) / (k_h \text{ derived from Eq.}(6.1.1) \text{ by using } b_2) \)

Bigger value of these two shall be the column group effect \( \mu_1 \).

\[ \mu_1 = \max (\mu_1^*, \mu_1^*) \]

When \( d_1 \geq 3b_1, d_2 < 3b_2 \), Fig.6.1.2(b)

\[ \mu_2 = 1 - 0.3(3 - R_2) \]

\( R_2 \): Pitch of improved column \( d_2 \)/width of improved column \( b_2 \) \((<3)\)

When \( d_1 < 3b_1, d_2 < 3b_2 \), Fig. 6.1.1(c)

\[ \mu_{12} = \mu_1 \cdot \mu_2 \]
c) Calculation of Young's modulus of improved column

Young's modulus of improved column $E_p$ may be obtained from design standard strength $F_c$.

$$E_p = 180 F_c \text{ (k N/㎡)}$$

When relation of (Young's modulus/unconfined compressive strength) is obtained through compressive test to the object improved column, the value shall be estimated to be Young's modulus. If deformation is controlled by average Young's modulus, calculated deformation derived from Young's modulus based on design standard strength shall be bigger than actual. But from the reasons that accuracy of equation to calculate deformation and deformation in improved column itself is small, above calculation method is practically available. It is necessary at any rate to examine accumulative data from now on deformation of improved ground.

d) Comparison of bending stress and allowable stress

Bending stress shall be calculated from load share of improved column in a), horizontal ground reaction force factor in b) and Young's modulus of improved column in c). It shall be recognized that bending stress shall be smaller than allowable stress.

① Calculation of bending moment

Bending moment is necessary to calculate bending stress and it is shown in Eq.(6.1.3).

$$M_p = \max (M_{\text{ACT}}, M_0)$$

$$M_{\text{ACT}} = \left( \frac{Q_p}{2 \beta} \right) \cdot R_{\text{MAX}}$$

$$M_0 = \left( \frac{Q_p}{2 \beta} \right) \cdot R_{\text{D0}}$$

(6.1.3)
$Q_p$: horizontal force to object improved column (kN)

$M_d$: bigger one of $M_{\text{max}}$ and $M_0$ (kN·m)

$M_{\text{max}}, M_0$: maximum bending moment in ground, and at top of the column (kN·m)

$R_{M_{\text{max}}}, R_{M_0}$: Factors of maximum bending moment in ground, and at the top of column respectively

$R_{M_{\text{max}}}$ and $R_{M_0}$ are shown in Table 6.1.2.

$$\beta = \frac{4}{k} \cdot k \cdot b_i/(4E_p \cdot I_p) \quad (\text{m}^{-1})$$

$k_h$: horizontal ground reaction factor (kN/m²)

$b_i$: width of improved column to horizontal force (m)

$E_p$: Young’s modulus of improved column (kN/㎡)

$I_p$: section secondary moment of improved column (m⁴)

when non-lapping: $(\pi/64) \cdot B^4$ ($B$: diameter of improved column)

when lapping: refer (4)

Factors relating to $M_{\text{max}}$ and $M_0$ depend on Table 6.1.2. Fixed ratio at column top $\alpha_r$ is 0.25 when there is rigid foundation slab on improved column. This is a factor that rotation of top portion of foundation column is fixed in some extent by foundation footing. This factor is used even if the gravel with the thickness of 10 to 20cm is spread between foundation slab and improved column. When factor is derived from appropriate test, the value can be used.

As for foundation beam just above improved column, safety of foundation beam shall be recognized by taking into account moment yielding at the top of improved column.

② Comparison of edge stress and allowable stress by bending moment

Edge stress shall be derived from following procedure and compared with allowable stress. Examination of edge stress is derived from Eq.(6.1.4) when non-lapping arrangement(①, ②, ⑤ in Table 6.1.1), and uniform load to foundation base in spite of lapping arrangement(③, ④).
Table 6.1.2 Factor for column calculation by linear elastic ground reaction force method

<table>
<thead>
<tr>
<th>Z</th>
<th>( R_{\text{M}_{\text{max}}} )</th>
<th>( R_M )</th>
<th>( R_{\text{l}\text{m}} )</th>
<th>( R_{yo} )</th>
<th>( R_{\text{M}_{\text{max}}} )</th>
<th>( R_M )</th>
<th>( R_{\text{l}\text{m}} )</th>
<th>( R_{yo} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.035</td>
<td>0.242</td>
<td>0.335</td>
<td>4.548</td>
<td>0.192</td>
<td>0.0</td>
<td>0.211</td>
<td>6.010</td>
</tr>
<tr>
<td>0.6</td>
<td>0.046</td>
<td>0.281</td>
<td>0.392</td>
<td>3.829</td>
<td>0.230</td>
<td>0.0</td>
<td>0.253</td>
<td>5.016</td>
</tr>
<tr>
<td>0.7</td>
<td>0.061</td>
<td>0.312</td>
<td>0.443</td>
<td>3.333</td>
<td>0.268</td>
<td>0.0</td>
<td>0.295</td>
<td>4.312</td>
</tr>
<tr>
<td>0.8</td>
<td>0.083</td>
<td>0.331</td>
<td>0.486</td>
<td>2.977</td>
<td>0.306</td>
<td>0.0</td>
<td>0.337</td>
<td>3.789</td>
</tr>
<tr>
<td>0.9</td>
<td>0.111</td>
<td>0.340</td>
<td>0.525</td>
<td>2.712</td>
<td>0.343</td>
<td>0.0</td>
<td>0.378</td>
<td>3.388</td>
</tr>
<tr>
<td>1.0</td>
<td>0.145</td>
<td>0.399</td>
<td>0.559</td>
<td>2.509</td>
<td>0.379</td>
<td>0.0</td>
<td>0.419</td>
<td>3.075</td>
</tr>
<tr>
<td>1.1</td>
<td>0.184</td>
<td>0.331</td>
<td>0.591</td>
<td>2.350</td>
<td>0.414</td>
<td>0.0</td>
<td>0.460</td>
<td>2.827</td>
</tr>
<tr>
<td>1.2</td>
<td>0.225</td>
<td>0.319</td>
<td>0.623</td>
<td>2.220</td>
<td>0.448</td>
<td>0.0</td>
<td>0.499</td>
<td>2.628</td>
</tr>
<tr>
<td>1.3</td>
<td>0.267</td>
<td>0.305</td>
<td>0.654</td>
<td>2.113</td>
<td>0.480</td>
<td>0.0</td>
<td>0.537</td>
<td>2.468</td>
</tr>
<tr>
<td>1.4</td>
<td>0.307</td>
<td>0.292</td>
<td>0.685</td>
<td>2.024</td>
<td>0.510</td>
<td>0.0</td>
<td>0.574</td>
<td>2.341</td>
</tr>
<tr>
<td>1.5</td>
<td>0.344</td>
<td>0.280</td>
<td>0.717</td>
<td>1.955</td>
<td>0.538</td>
<td>0.0</td>
<td>0.610</td>
<td>2.239</td>
</tr>
<tr>
<td>1.6</td>
<td>0.377</td>
<td>0.270</td>
<td>0.748</td>
<td>1.893</td>
<td>0.563</td>
<td>0.0</td>
<td>0.643</td>
<td>2.159</td>
</tr>
<tr>
<td>1.7</td>
<td>0.407</td>
<td>0.262</td>
<td>0.778</td>
<td>1.845</td>
<td>0.585</td>
<td>0.0</td>
<td>0.673</td>
<td>2.098</td>
</tr>
<tr>
<td>1.8</td>
<td>0.432</td>
<td>0.256</td>
<td>0.807</td>
<td>1.808</td>
<td>0.604</td>
<td>0.0</td>
<td>0.701</td>
<td>2.051</td>
</tr>
<tr>
<td>1.9</td>
<td>0.453</td>
<td>0.252</td>
<td>0.834</td>
<td>1.779</td>
<td>0.620</td>
<td>0.0</td>
<td>0.726</td>
<td>2.018</td>
</tr>
<tr>
<td>2.0</td>
<td>0.471</td>
<td>0.249</td>
<td>0.858</td>
<td>1.759</td>
<td>0.632</td>
<td>0.0</td>
<td>0.747</td>
<td>1.994</td>
</tr>
<tr>
<td>2.1</td>
<td>0.484</td>
<td>0.247</td>
<td>0.880</td>
<td>1.744</td>
<td>0.642</td>
<td>0.0</td>
<td>0.765</td>
<td>1.979</td>
</tr>
<tr>
<td>2.2</td>
<td>0.494</td>
<td>0.246</td>
<td>0.898</td>
<td>1.735</td>
<td>0.649</td>
<td>0.0</td>
<td>0.779</td>
<td>1.970</td>
</tr>
<tr>
<td>2.3</td>
<td>0.501</td>
<td>0.246</td>
<td>0.913</td>
<td>1.730</td>
<td>0.653</td>
<td>0.0</td>
<td>0.789</td>
<td>1.966</td>
</tr>
<tr>
<td>2.4</td>
<td>0.505</td>
<td>0.246</td>
<td>0.925</td>
<td>1.728</td>
<td>0.656</td>
<td>0.0</td>
<td>0.796</td>
<td>1.965</td>
</tr>
<tr>
<td>2.5</td>
<td>0.508</td>
<td>0.246</td>
<td>0.933</td>
<td>1.728</td>
<td>0.657</td>
<td>0.0</td>
<td>0.801</td>
<td>1.967</td>
</tr>
<tr>
<td>2.6</td>
<td>0.509</td>
<td>0.247</td>
<td>0.939</td>
<td>1.729</td>
<td>0.658</td>
<td>0.0</td>
<td>0.803</td>
<td>1.971</td>
</tr>
<tr>
<td>2.7</td>
<td>0.509</td>
<td>0.247</td>
<td>0.942</td>
<td>1.732</td>
<td>0.656</td>
<td>0.0</td>
<td>0.803</td>
<td>1.975</td>
</tr>
<tr>
<td>2.8</td>
<td>0.507</td>
<td>0.248</td>
<td>0.944</td>
<td>1.734</td>
<td>0.655</td>
<td>0.0</td>
<td>0.802</td>
<td>1.979</td>
</tr>
<tr>
<td>2.9</td>
<td>0.506</td>
<td>0.248</td>
<td>0.943</td>
<td>1.737</td>
<td>0.653</td>
<td>0.0</td>
<td>0.800</td>
<td>1.984</td>
</tr>
<tr>
<td>3.0</td>
<td>0.504</td>
<td>0.249</td>
<td>0.943</td>
<td>1.740</td>
<td>0.652</td>
<td>0.0</td>
<td>0.798</td>
<td>1.988</td>
</tr>
<tr>
<td>3.2</td>
<td>0.501</td>
<td>0.249</td>
<td>0.939</td>
<td>1.745</td>
<td>0.649</td>
<td>0.0</td>
<td>0.793</td>
<td>1.994</td>
</tr>
<tr>
<td>3.4</td>
<td>0.498</td>
<td>0.250</td>
<td>0.944</td>
<td>1.748</td>
<td>0.647</td>
<td>0.0</td>
<td>0.790</td>
<td>1.998</td>
</tr>
<tr>
<td>3.6</td>
<td>0.496</td>
<td>0.250</td>
<td>0.932</td>
<td>1.750</td>
<td>0.645</td>
<td>0.0</td>
<td>0.787</td>
<td>2.001</td>
</tr>
<tr>
<td>3.8</td>
<td>0.495</td>
<td>0.250</td>
<td>0.930</td>
<td>1.750</td>
<td>0.645</td>
<td>0.0</td>
<td>0.788</td>
<td>2.001</td>
</tr>
<tr>
<td>4.0</td>
<td>0.495</td>
<td>0.250</td>
<td>0.928</td>
<td>1.751</td>
<td>0.644</td>
<td>0.0</td>
<td>0.785</td>
<td>2.002</td>
</tr>
<tr>
<td>4.2</td>
<td>0.495</td>
<td>0.250</td>
<td>0.928</td>
<td>1.751</td>
<td>0.644</td>
<td>0.0</td>
<td>0.785</td>
<td>2.001</td>
</tr>
<tr>
<td>4.4</td>
<td>0.495</td>
<td>0.250</td>
<td>0.927</td>
<td>1.751</td>
<td>0.644</td>
<td>0.0</td>
<td>0.785</td>
<td>2.001</td>
</tr>
<tr>
<td>4.6</td>
<td>0.495</td>
<td>0.250</td>
<td>0.927</td>
<td>1.750</td>
<td>0.645</td>
<td>0.0</td>
<td>0.785</td>
<td>2.000</td>
</tr>
<tr>
<td>4.8</td>
<td>0.495</td>
<td>0.250</td>
<td>0.925</td>
<td>1.750</td>
<td>0.645</td>
<td>0.0</td>
<td>0.785</td>
<td>2.000</td>
</tr>
<tr>
<td>5.0</td>
<td>0.495</td>
<td>0.250</td>
<td>0.928</td>
<td>1.750</td>
<td>0.645</td>
<td>0.0</td>
<td>0.785</td>
<td>2.000</td>
</tr>
</tbody>
</table>
\[
\sigma_{\max} = \frac{q}{a_p} + \frac{M_d}{(2I_p/b_2)} \\
\sigma_{\min} = \frac{q}{a_p} - \frac{M_d}{(2I_p/b_2)} \leq \text{allowable compressive stress } f_c \\
\sigma_{\min} = \frac{W_p}{A_p} - \frac{M_d}{(2I_p/b_2)} \geq \text{allowable tensile stress } f_t
\]

\[
(6.1.4)
\]

\(\sigma_{\max}, \sigma_{\min}\) : edge stress at compressive side and tensile side respectively \((\text{kN/m}^2)\)

\(q\) : average stress at foundation base \((\text{kN/m}^2)\)

\(W_p\) : vertical load to object improved column \((\text{kN})\)

\(f_c, f_t\) : allowable compressive stress, and allowable tensile stress, shown in Table 6.1.3 \((\text{kN/m}^2)\)

\(b_2\) : improved width to horizontal force \((\text{m})\)

\(A_p\) : cross sectional area of improved column \((\text{m}^2)\)

\(a_p\) : improved ratio

(in case of ⑤ in Table 6.1.1, maximum stress to foundation base \(q_{\max}\) is used instead of \(q\)).

When load of foundation base is non-uniform in lapped arrangement (⑥, ⑦, ⑧ in Table 6.1.1), edge stress shall be calculated at each position as shown in Fig. 6.1.3 and Fig. 6.1.4 taking into account both maximum bending moment in ground and bending moment at the top of column. The equation is (6.1.5).

\[
\sigma_{\max} \leq \text{allowable compressive stress } f_c \\
\sigma_{\min} \geq \text{allowable tensile stress } f_t
\]

(6.1.5)

Stress shall be examined at the edge because stress at the edge is maximum or minimum when load to foundation base is trapezoid as in Fig. 6.1.4.

Allowable compressive stress and allowable tensile stress are defined to design standard strength \(F_c\) as shown in Table 6.1.3. \(F_c\) is value under condition that compressive stress functions uniformly to whole cross section, but is compared with the edge stress here. The value shown in Table 6.1.3 will be safety side.
Also, upper value limit of allowable tensile stress is fixed with reason that value of (tensile strength/unconfined strength) will be smaller proportion to bigger of unconfined compressive strength. Another reason is that it should not expect much tensile strength of improved soil. Relation between tensile strength and unconfined compressive strength is shown in Fig. 6.1.5 and Table 6.1.3. Fixing of Table 6.1.3 seems to be proper from the figure.

Improved lapping columns resist to external force as one block, but the block must be checked to bear shearing force described in (2) later. ③ in Table 6.1.1 will be simplified to ⑦ in the same table with reason that horizontal surface can bear horizontal force. Also it is possible to calculate independently to horizontal and vertical force.

<table>
<thead>
<tr>
<th>Table 6.1.3  Allowable compressive stress and allowable tensile stress</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static state</strong></td>
</tr>
<tr>
<td>Allowable compressive stress $f_c$</td>
</tr>
<tr>
<td>Allowable tensile stress $f_t$</td>
</tr>
</tbody>
</table>

$F_c$ : design standard strength, $q_{utmax} = 300 \text{kN/m}^2$
Occurrence depth of the maximum bending moment in the ground shall be calculated by Eq.(6.1.6) as a reference.

$$l_m = \frac{1}{\beta} \cdot R_{1m}$$  \hspace{1cm} (6.1.6)

$R_{1m}$: factor on occurrence depth of the maximum bending moment in ground [Table(6.1.2)].

(2) Examination of Shearing Stress

a) Examination method

Shearing stress of improved column shall be checked by Eq.(6.1.7) considering shape of improved column.

$$\tau_{\text{max}} = \kappa \cdot \bar{\tau} = \kappa \cdot \left( \frac{Q_p}{A_p} \right) \leq f_\tau$$  \hspace{1cm} (6.1.7)

$\tau_{\text{max}}$: maximum shearing stress in section considering shape of column

$\bar{\tau}$: average shearing stress

$f_\tau$: allowable shearing stress(6.1.8)
Allowable bearing stress shall be derived from Eq.(6.1.8).

\[
\begin{align*}
  &\text{when static state } \tau = 1/3 \cdot F_	ext{c} = 1/3 \cdot \min \{ 0.3 F_	ext{c} + (Q_p/A_p) \tan \phi, 0.5 F_	ext{c} \} \\
  &\text{when middle scale earthquake } \tau = 2/3 \cdot F_	ext{c} = 2/3 \cdot \min \{ 0.3 F_	ext{c} + (Q_p/A_p) \tan \phi, 0.5 F_	ext{c} \} \\
\end{align*}
\]

\[(6.1.8)\]

\(F_c\): design shearing stress \((\text{kN}/\text{㎡})\)

\(\sigma_n\): normal stress to shearing section \((\text{kN}/\text{㎡})\)

supposed that normal stress corresponding to horizontal average shearing force acts to vertical shearing section \(\sigma_n = (Q_p/A_p)\)

\(Q_p\): horizontal force to block of improved column\((\text{refer Table 6.1.1})\) \((\text{kN})\)

\(A_p\): area of block of improved column\((\text{neglect improved column outside of foundation base})\) \((\text{㎡})\)

\(\phi\): angle of internal friction\(30^\circ\)

\(\kappa\): shape factor\((\text{refer following b)})\)

**b) Determination of shape factor**

1. when column is non-lapping arrangement\((\text{refer Table 6.1.1 ①, ②, ⑤})\)

In case of non-lapping arrangement, since examination is carried out for one improved column, supposed that shape is circular, shape factor is \(\kappa = 4/3\) .

2. when column is lapping arrangement and when it is improved as wall type or block type\((\text{refer Table 6.1.1 ④, ⑦, ⑧})\)

Shearing stress of section \(X—X’\) in general section is

\[
\tau_{x-x’} = Q \cdot \frac{S_x}{b_x \cdot I} 
\]

Average shearing stress is \(\bar{\tau} = Q/A\) . Therefore, shape factor \(\kappa\) is expressed as (6.1.9)

\[
\kappa = \left( \frac{\tau_{x-x’}}{\bar{\tau}} \right)_{\text{max}} = \left( \frac{Q \cdot \frac{S_x}{b_x \cdot I}}{Q/A} \right)_{\text{max}} = \left( \frac{S_x}{b_x \cdot I} \right)_{\text{max}} \cdot A 
\]

\[(6.1.9)\]

\(\tau_{x-x’}\): shearing stress in section \(X—X’\) \((\text{kN}/\text{㎡})\)

\(Q\): horizontal force \((\text{kN})\)

\(A\): area \((\text{㎡})\)
\( b x \): width in section \( X—X' \) (refer Fig. 6.1.6) \((m)\)
\( b \): width of column, \( t \): width of lapping
\[ b_L = b \cdot \sin \theta, \quad \theta = \cos^{-1}(1 - t/b) \]
\( Sx \): sectional primary moment section outside of section \( X—X' \) to neutral axis \((m^3)\)
\( I \): sectional secondary moment of whole section \((m^4)\)

Since \( S_x \) is maximum in neutral axis in general, \( \tau \) is maximum in this position. In case of lapping arrangement width in lapped portion \( x \) is minimum, and it is necessary to decide the shape factor considering above fact.

![Fig. 6.1.6 Concept of shape factor](image)

Calculation result of shape factor is shown in Fig. 6.1.7 when lapping width ratio (lapping width \( t \) /improved column diameter \( b \)) and number of improved column are variable. When column number is even, lapping position and neutral axis position coincide, hence shearing stress will be the maximum at this position. When column number is odd, above both do not coincide and shape factor in lapping portion will be large. The smaller of even column number and the more of odd column number, shape factor will be larger.
The shape factor is shown in Table 6.1.4 for the case of two columns. This is upper value, and it will be safety side. Lapping width is in general 10 to 20% of column diameter from work execution and quality of lapping portion point of view. Fig 6.1.7 will be used in order to obtain accurate shape factor to column number. In this case, shape factor is case of parallel lapping to horizontal force. At the same time, shape factor in case of right angle lapping to the horizontal force is smaller than non-lapping case because only few defects occur at the lapping portion. At any rate Fig. 6.1.7 is in safety side.

③ When lapping arrangement is improved in grid state

In case of Table 6.1.1③, shape factor may be derived from Eq.(6.11.9). Also

Table 6.1.4  Shape factor of lapped column (for two columns)

<table>
<thead>
<tr>
<th>(lapped width) / (improved column diameter) (%)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape factor κ</td>
<td>4.1</td>
<td>3.0</td>
<td>2.5</td>
<td>2.2</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Fig.6.1.7  Relation between lapped width ratio and shape factor
Eq.(6.1.10) may be used as approximate value for this purpose.(refer Fig.6.1.8)

$$\kappa = \kappa_1 / \lambda$$  \hspace{1cm} (6.1.10)

\(\lambda\) : (lapping length) / (column width)
in Fig. 6.1.8 \(\lambda = b_L / (d/2)\)

\(\kappa_1\) : shape factor of grid state shown in Fig.6.1.8 of I type section
Shape factor of grid section shown in Fig.6.1.8 (a) shall be replaced to equivalent I type section shown in (b). \(\tau_{\text{max}}\) will be derived by following equation.

$$\tau_{\text{max}} = \frac{3}{2} \frac{Q}{B \cdot D} \frac{d^2 + B(D^2 - d^2)/t}{B \cdot d^2 - (B - t) \cdot d^2}$$

Average shearing strength is calculated by \(\overline{\tau} = \frac{Q}{B \cdot D - d (B - t)}\)

(Fig.6.1.8 Reference figures of shape factor in grid type)

Therefore, shape factor \(\kappa_1\) in grid or I type section will be Eq.(6.1.11).

$$\kappa_1 = \frac{\tau_{\text{max}}}{\overline{\tau}} = \frac{3}{2} \frac{B \cdot D - d (B - t)}{B \cdot D} \frac{d^2 + B(D^2 - d^2)/t}{B \cdot d^2 - (B - t) \cdot d^2}$$  \hspace{1cm} (6.1.11)

Equation (6.1.10) is the shape factor of grid or I type section based on Eq.(6.1.9) taking into section defect in lapping portion.

④ When wall type arranged right angle to horizontal force
This is a case shown in Fig.6.1.13 (a), improved column is arranged right angle to horizontal force. Shape factor \(\kappa\) in this case is 4/3 applied correspondingly.
c) Examination of shearing stress shall be carried out by using finite element method considering section defects in lapping portion except above method. When detailed examination is carried out taking into account the effect of peripheral ground, shearing stress will be calculated smaller than above mention method.

(3) Transformation at the Top of Column (Pile)
Displacement at the top of column (pile) shall be shown in Eq.(6.1.12).

\[ y_0 = \frac{Q_p}{4E_p\cdot I_p\cdot \beta^2} R_{yo} \]  

(6.1.12)

\( y_0 \): Displacement at the top of column (pile)  
\( Q_p \): horizontal load to the object column  
\( \beta \): \[ \beta = \sqrt[4]{\frac{k_s \cdot b \cdot i}{(4E_p \cdot I_p)}} \] (m\(^{-1}\))

\( k_s \): horizontal ground reaction force factor (kN/m\(^3\))
\( b \): width of improved body in right angle direction to horizontal force (m)
\( E_p \): Young’s modulus of improved column
\( I_p \): sectional secondary moment of improved column

when non-lapping arrangement \( \left( \frac{\pi}{64} \right) \cdot B^4 \) (B is diameter)
when lapping, refer (4)
\( R_{yo} \): factor on transformation at the top of column (pile) (Table 6.1.2)

(4) Sectional Constant in Case of Lapping Arrangement
Sectional primary and secondary moments of \( x' - x' \) axis after displacement \( x_0 \) from \( x - x \) axis shall be obtained by following procedures.(refer Fig.6.1.9).

Fig.6.1.9 Cross sectional factor in case of axis movement
Sectional primary moment \( S' = \int (x-x_0)\,dA \quad = \int xdA - \int x_0\,dA = S - x_0 \cdot A \)

Sectional secondary moment \( I' = \int (x-x_0)^2\,dA \quad = \int x^2\,dA - 2x_0 \int xdA + x_0^2\int dA \quad = I - 2x_0 \cdot S + x_0^2 \cdot A \)

In case of lapping, sectional constant of lapping arrangement consisting of plural columns can be obtained from sectional primary moment \( S \), sectional secondary moment \( I \) and area \( A \) in parallel and right angle direction to \( x-x \) axis. Fig.6.1.10 shows an example consisting of three columns.

Table 6.1.5 shows calculation equation of sectional constant, and Table 6.1.6 shows value of sectional constant in case of column diameter 100 and 60cm. Sectional moment

(a) Lapped in one direction  
(b) Lapped in two direction

Fig.6.1.10  Shape factor in case of lapping arrangement

| No | Form | Formul
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>( I_0 = \int x^2 \sqrt{b^2 - 4 \left(\frac{x + \frac{b - t}{2}\right)^2},dx )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( S_0 = \int x \sqrt{b^2 - 4 \left(\frac{x + \frac{b - t}{2}\right)^2},dx )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( A_0 = \frac{b^2}{4} - \left(\frac{b - t}{2}\right)^2 \tan \theta )</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>( I_0 = I_0 \quad S_0 = -S_0 \quad A_0 = A_0 )</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( I_0 = 2 \cdot I_0 \quad S_0 = 0 \quad A_0 = 2 \cdot A_0 )</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>( I_0 = \pi \int_0^{\frac{\sqrt{b^2 - 4 \left(\frac{x + \frac{b - t}{2}\right)^2}}}{2}} x^2 \left(\frac{b^2 - 4 \left(\frac{x + \frac{b - t}{2}\right)^2}{2}\right),dx )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( S_0 = 0 \quad A_0 = A_0 )</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>( I_0 = 2 \cdot I_0 \quad S_0 = 0 \quad A_0 = 2 \cdot A_0 )</td>
</tr>
</tbody>
</table>

\( \theta = \cos^{-1}\left(1 - \frac{k}{b}\right) \) (rad)
and sectional factor for examination of vertical bearing stress and bending stress can be examined by modified rectangle from round improved column. In this case, area must be the same. The concept of modified rectangle is shown in Fig.6.1.11.

![Diagram of Modified Rectangle and Improved Column](image)

\[
b_1 = \alpha \cdot b_1 \\
b_2 = \alpha \cdot b_2 \\
\alpha \cdot b_1 \times \alpha \cdot b_2 = A_0
\]

**Table 6.1.6 Cross sectional constant of basic figure**

(a) Improved column diameter of 100 cm

<table>
<thead>
<tr>
<th>Lapped width</th>
<th>L</th>
<th>( I (m^4) \times 10^8 )</th>
<th>( S (m^2) \times 10^6 )</th>
<th>( A (m^2) \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 (m)</td>
<td>1</td>
<td>75.0</td>
<td>75.0</td>
<td>149.9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>52.4</td>
<td>-52.4</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>52.3</td>
<td>52.3</td>
<td>104.6</td>
</tr>
</tbody>
</table>

(b) Improved column diameter of 60 cm

<table>
<thead>
<tr>
<th>Lapped width</th>
<th>L</th>
<th>( I (m^4) \times 10^8 )</th>
<th>( S (m^2) \times 10^6 )</th>
<th>( A (m^2) \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 (m)</td>
<td>1</td>
<td>57.9</td>
<td>57.9</td>
<td>115.8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>40.5</td>
<td>40.5</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>40.3</td>
<td>40.3</td>
<td>80.6</td>
</tr>
</tbody>
</table>
(5) Design of Grid and Wall Types (mainly for continuous foundation)

In case of Table 6.1.1 ⑥, when $L/b$ (shown in Fig.6.1.12) becomes large, since there is difference in horizontal resistance to improved column between parallel and right angle to function force, it is difficult to judge that improved column ABCD behaves as a body. The following way of thinking will be the safe side.

① Case of Fig.6.1.12 (a) case 1

Stress shall be checked to the shared load by setting the domain of improved column according to column arrangement for right angle improved column to force (dark portion in the Fig.). Improved column in parallel to force shall bear all the horizontal force corresponding to column load above improved column. Vertical load shall be load distribution on foundation footing base.

② Case of Fig.6.1.12 (b) case 2

Horizontal force shall be born by improved column in parallel direction. Vertical load shall be load distribution on foundation footing base, and it shall be born by parallel
and right angle improved column to force. The state as Fig.6.1.12 is arisen when improvement is carried out under continuous foundation.